Proceedings of the 2000 IEEE/RSJ International Conference on Intelligent Robots and Systems

Robust Control of Cascaded Nonlinear Systems

Jong Hyeon Park and Jongguk Yim
Mechatronics Lab
School of Mechanical Engineering
Hanyang University
Seoul, 133-791, Korea
email: jongpark@email.hanyang.ac.kr

Abstract

Robust control is designed for nonlinear uncertain systems, which can be decomposed into two cascaded subsystems, that is, a series connection of two nonlinear subsystems. For such system, a recursive design is used to include second subsystem into robust control. The recursive design procedure is constructive and contains two steps. First, fictitious robust controller for first subsystem is designed as if the subsystem had an independent control. As the fictitious control, nonlinear H_∞ control using the energy dissipation is designed in the sense of L_2-gain attenuation from the disturbance caused by uncertainties to performance. Second, the actual robust control is designed recursively by Lyapunov's second method. The designed robust control is applied to robotic system with actuators, in which physical control inputs are not the torque to robot links but electrical signals to actuators.

1 Introduction

For a class of nonlinear system, in which every system is a series connection of finite number of nonlinear subsystem, the recursive design is applied to design stabilizing control. Interesting progress in the recursive design has been achieved in adaptive control of feedback linearizable systems [1].

Since many systems inherently have uncertainties such as parameter variations, external disturbances, and unmodelled dynamics, the robust control is considered in the recursive design. To design robust controllers, it is usual to use Lyapunov's second method, as proceeded in the existing results [2, 3]. One of the difficulties is that Lyapunov's second method requires a Lyapunov function for control design.

Another robust control, which has attracted attention of many researchers, is H_∞ control. Although the nonlinear H_∞ control has been derived by the L_2-gain analysis based on the concept of the energy dissipation [4, 5], its applications are not easy due to the solution to Hamilton Jacobi inequality (HJ inequality). The HJ inequality is a first-order partial differential inequality and it is difficult to obtain its solution. H_∞ control problem in nonlinear systems reduces to the existence of the solution to HJ inequality and many methods are proposed in recent papers [6, 7, 8, 9].

In present paper, robust control is designed for cascaded nonlinear uncertain system using recursive design which is composed of two steps. In first step, fictitious robust controller for first subsystem is designed as if the subsystem had independent control. As the fictitious control, the nonlinear H_∞ control is used in this work to guarantee the performance of system and the differentiability of the control, which is required to design the actual control recursively. The solution to HJ inequality can be obtained through a more tractable nonlinear matrix inequality (NLMI) method and the fact that the matrices forming the NLMI is bounded [9]. In second step, the actual robust control is designed recursively by Lyapunov's second method.

The designed control is applied to robotic system with actuators. In a block diagram representation of the overall robotic system, the two blocks representing actuator dynamics and robot dynamics, respectively, are connected in a series.

This paper is organized as follow. In section 2, the recursive design procedures are presented for certain and uncertain system. In section 3, the robust control is designed for the robotic system with actuators using the procedures in section 2. In section 4, simulations are performed to confirm the robust performance of the proposed controller for robot manipulator under parameter uncertainty. In section 5, the conclusion is presented.

2 Recursive Design

2.1 Certain System

The class of nonlinear system considered in this paper consists of those which is the series connection of two subsystems and whose dynamics are described by

$$\begin{aligned} \dot{x}_1 &= f(x_1) + g(x_1)\varphi(x_2) & (1) \\ \dot{x}_2 &= Ax_2 + Bu + h(x_1), & (2) \end{aligned}$$

where x_1 and $x_2 \in \mathbb{R}^n$ are the states of systems and $u \in \mathbb{R}^m$ is a control input, and A and B are constant matrices. Eq. (2) of the second subsystem is a differential equation whose output is the input signal to the first subsystem. In a block diagram representation of the overall system, the two blocks are connected in a series. The recursive design exploits this structure property.

In recursive design, it is required that there exists the fictitious control, which stabilize the first subsystem, Eq. (1).

Assumption 1 *(Global Stabilizability) There exists a C^1 control law $K(x_1)$ such that the equilibrium $x_1 = 0$ of the system $\dot{x}_1 = f(x_1) + g(x_1)K(x_1)$ is globally stable. This is established with a C^2 positive definite function $E(x_1)$ such that*

$$\frac{\partial E}{\partial x_1}\left(f(x_1) + g(x_1)K(x_1)\right) \leq 0.$$

In the first subsystem, this control law is not implementable and its effect must be achieved through the x_2-subsystem.

Assumption 2

1. $\varphi(x_2) = K_w x_2$
2. *There exists a fictitious state x_w such that $K(x_1) = K_w x_w$.*

Theorem 1 *If Assumption 1 and 2 are satisfied with the fictitious state x_w then the overall system is feedback passive with respect to the new state*

$$y = x_2 - x_w \tag{3}$$

and its passivity is achieved with the feedback control

$$u = B^{-1}\left(-Ax_2 - h(x_1) - \dot{x}_w - K_w^T(L_gE)^T + r\right). \tag{4}$$

proof: Using Eq. (3) as a new coordinate, we rewrite Eqs. (1) and (2) as

$$\begin{aligned} \dot{x}_1 &= f(x_1) + g(x_1)K_w(x_w + y) & (5) \\ \dot{y} &= Ax_2 + Bu + h(x_1) + \dot{x}_w. & (6) \end{aligned}$$

To show that the feedback control achieves passivity, we use the positive definite storage function

$$V(x_1, y) = E(x_1) + \frac{1}{2}y^Ty.$$

With Eq. (5) and (6), its time-derivative is

$$\begin{aligned} \dot{V} &= \frac{\partial E}{\partial x_1}\left(f(x_1) + g(x_1)K_w x_w\right) \\ &+ \; y^T\left(K_w^T(L_gE)^T + Ax_2 + Bu + h(x_1) + \dot{x}_w\right). \end{aligned}$$

By *assumptions* 1 and 2, the feedback control law, Eq. (4) proves passivity. Q.E.D.

With the additional feedback $r = -ky$, $k > 0$ its derivative is $\dot{V} \leq -ky^Ty$. This proves global stability of its equilibrium $(x_1, y) = (0, 0)$.

2.2 Uncertain System

The recursive design in the previous section can be applied to a class of nonlinear systems, which have uncertainties and require good tracking performance. Before proceeding with a detail recursive design, the following Assumption 3 is needed to obtain the fictitious control easily when the nonlinear H_∞ control is used as the fictitious control.

Assumption 3 *There exist a C^1 control law $K(x_1, x_{1d}, u) = K'(x_1, x_{1d}) + u$ such that the system $\dot{x} = f(x_1) + g(x_1)K(x_1, x_{1d}, u)$ is transformed to*

$$\dot{s} = F(x_1)s + G_1(x_1)w_1 + G_2(x_1)u,$$

where $x_{1d} \in \mathbb{R}^n$ is the desired trajectory, $s(x_1, x_{1d}) \in \mathbb{R}^n$ is the modified state, $w_1 \in \mathbb{R}^w$ is the disturbance caused by uncertainties and $u \in \mathbb{R}^m$ is the control input for the robustness.

2.2.1 Fictitious Control

In the presence of disturbance, robust control is needed as the fictitious control. In order to guarantee the robustness for disturbance and the differentiability of the control, which is required to design the actual control recursively, the nonlinear H_∞ control is designed as the fictitious control. To find the nonlinear H_∞ control is to find a stabilizing state-feedback control input such that the closed-loop system has a L_2-gain equal to or less than γ in the input-to-output sense. In nonlinear H_∞ control design, it is essential to find the solution to the associated Hamilton-Jacobi (HJ) inequality derived from input-output energy dissipation. If a solution exists, then it will guarantee the stability as well as the disturbance attenuation.

By *Assumption 3* and with the performance vector z, the first nonlinear subsystem, Eq. (1) can be described as

$$\begin{aligned} \dot{s} &= F(x_1)s + G_1(x_1)w_1 + G_2(x_1)u \\ z &= Hs + Du,\ H^T D = 0,\ D^T D > 0, \end{aligned} \tag{7}$$

where $F(x_1)$, $G_1(x_1)$, $G_2(x_1)$, H, D are C^0 matrix-valued function of suitable dimensions.

In the form of Eq. (7), the derived HJ inequality is more tractable. The design of a nonlinear H_∞ controller for the nonlinear system in the form of Eq. (7) is summarized as the following theorem [9].

Theorem 2 *Given $\gamma > 0$, suppose there exists a C^0 matrix-valued function P satisfying*

$$\begin{aligned} &P^T F(x_1) + F^T(x_1)P + \frac{1}{\gamma^2}P^T G_1(x_1)G_1^T(x_1)P \\ &+H^T H - P^T G_2(x_1)[D^T D]^{-1}G_2^T(x_1)P \leq 0 \end{aligned} \tag{8}$$

and there exists a non-negative function $E(s) \geq 0$ such that $\partial E/\partial s = 2s^T P^T$. Then the control input satisfying L_2-gain $\leq \gamma$ is

$$u = -[D^T D]^{-1}G_2^T(x)Ps$$

and the derivative of the storage function satisfies

$$\dot{E} \leq \gamma^2\|w\| - \|z\|^2.$$

To obtain the solution to the Eq. (8) easily, it is transformed to a nonlinear matrix inequality (NLMI) using the Schur complement

$$\begin{bmatrix} P^T F + F^T P + H^T H + \frac{1}{\gamma^2}P^T G_1 G_1^T P & P^T G_2 \\ G_2^T P & D^T D \end{bmatrix} \leq 0.$$

Solving the above NLMI yields convex optimization problem. Unlike the linear case, this convex problem is not finite dimensional. However, if the matrices forming the NLMI are bounded, then we only need to solve a finite number of LMIs.

The stabilizing control input at the first step becomes

$$K = K' - [D^T D]^{-1}G_2^T(x_1)Ps.$$

2.2.2 Real Control

Assume there exists the fictitious state x_{w1} and x_{w2} satisfying *Assumption 2*, that is,

$$\begin{aligned} K &= K' + u \\ &= K_w(x_{w1} + x_{w2}) \\ &= K_w x_w. \end{aligned}$$

It follows that the choice of $x_2 = x_w$, if permitted, would ensure robust stability. Since x_2 is not a control input, we cannot let $x_2 = x_w$. In the cascade, its effect must be achieved through the second subsystem.

Using $y = x_2 - x_w$ as a new state, we rewrite

$$\begin{aligned} \dot{s} &= F(x_1)s + G_1(x_1)w + G_2(x_1)K_w(y + x_{w2}) \\ \dot{y} &= Ax_2 + Bu + h(x_1) - \dot{x}_w. \end{aligned}$$

To design the robust control, which achieves L_2-gain property, we use the positive definite storage function

$$V(s,y) = E(s) + \frac{1}{2}y^T y.$$

Its time derivative is

$$\begin{aligned} \dot{V} &\leq \gamma^2\|w\|^2 - \|z\|^2 + y^T(2K_w^T G_2^T Ps \\ &\quad +Ax_2 + Bu + h(x_1) - \dot{x}_w) \\ &\leq \gamma^2\|w\|^2 - \|z\|^2 + y^T(\Delta A + Bu). \end{aligned}$$

ΔA can be bounded as

$$\begin{aligned} \|\Delta A\| &\leq \alpha_1\|s\| + \alpha_2\|x_2\| + \|h(x_1)\| + \|\dot{x}_{w1}\| + \|\dot{x}_{w2}\| \\ &:= \rho(s, x_1, x_2, \dot{x}_{w1}, \dot{x}_{w2}). \end{aligned}$$

If it is assumed that $\underline{B} \leq B \leq \overline{B}$ for known upper and lower bounding matrices, the control input can be chosen as

$$u = -\underline{B}^{-1}\left(\frac{y}{\|y\|}\rho - y\right).$$

With the designed control input, the storage function satisfies $\dot{V} \leq \gamma^2\|w\|^2 - \left(\|z\|^2 + \beta\|y\|^2\right)$, $\beta > 0$, which achieves L_2-gain property.

3 Actuator-Level Control for Robot Manipulator

3.1 Dynamics of Robot Manipulator with Actuators

Consider the following dynamics of robot manipulator actuated by permanent magnet DC machines.

$$M(q)\ddot{q} + N(q,\dot{q})\dot{q} + G(q) = K_\tau i \tag{9}$$

$$L_m \frac{di}{dt} + R_m i + K_m \dot{q} = v, \tag{10}$$

where $q \in \mathbb{R}^n$ is the joint position, $M(q) \in \mathbb{R}^{n\times n}$ is the positive definite symmetric inertia matrix, $N(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal and coriolis torque, $G(q) \in \mathbb{R}^n$ and represents the gravitational torque. Symbols R_m and L_m denote the resistance and inductance of the armature circuit; K_τ and K_m are torque and back emf parameter of the motor; i is the current in the armature of the motor; v is the armature voltage. Eq. (10) of actuator dynamics is a first-order differential equation whose output i is the input signal to robot dynamics.

3.2 Fictitious Control (Torque-Level Control)

3.2.1 Transformation of Dynamics

In an overall system representing actuator dynamics and robot dynamics, the two subsystems are connected in a series. That is, the overall system can be decomposed into two cascaded subsystems. The recursive design uses this structure.

Before proceeding with a detail recursive design, modified error for joint tracking, which satisfy *Assumption 3* is defined as

$$\begin{aligned} s &= \dot{q} - \{\dot{q}_d - \Lambda(q - q_d)\} \\ &= \dot{q} - \dot{q}_r, \end{aligned}$$

where q_d and $\dot{q}_d$ are the desired position and velocity respectively. If the elements of vector approach to zeros at $t \to \infty$, so does the tracking error of joint.

At the torque level of the robotic system, a suitable control input satisfying *Assumption 3* can be chosen as

$$K = \hat{M}(q)\ddot{q}_r + \hat{N}(q,\dot{q})\dot{q}_r + \hat{G}(q) + u. \tag{11}$$

Then, Eq. (9) is transformed to

$$\dot{s} = F(q,\dot{q})s + G_1(q)w + G_2(q)u, \tag{12}$$

where $F(q,\dot{q}) = -M^{-1}(q)N(q,\dot{q})$, $G_1 = M^{-1}(q)$, $G_2 = M^{-1}(q)$ and $w = \tilde{M}(q)\ddot{q}_r + \tilde{N}(q,\dot{q})\dot{q}_r + \tilde{G}$ denotes a disturbance vector caused by model uncertainties.

3.2.2 The Solution to HJ Inequality Using LMI

To derive the HJ inequality for the robot manipulator dynamics transformed to affine form, each matrix term of Eq. (12) is substituted into Eq. (8). Then

$$\begin{aligned} &-\left(MP^{-T}\right)^{-1} N - N^T \left(P^{-1}M^T\right)^{-1} + H^T H \\ &\quad + \frac{1}{\gamma^2}\left(MP^{-T}\right)^{-1}\left(P^{-1}M^T\right)^{-1} \\ &\quad - \left(MP^{-T}\right)^{-1}\left(D^T D\right)^{-1}\left(P^{-1}M^T\right)^{-1} < 0. \end{aligned}$$

Premultiplying and postmultiplying the inequality by the positive definite matrices MP^{-T} and $P^{-1}M^T$ respectively, then the HJ inequality becomes

$$\begin{aligned} &-NQM^T - MQ^T N^T + MQ^T H^T HQM^T \\ &\quad + \frac{1}{\gamma^2} I - (D^T D)^{-1} < 0, \end{aligned} \tag{13}$$

where $Q = P^{-1}$. Using the Schur complement, Eq. (13) can be described as an NLMI

$$\begin{bmatrix} -NQM^T - MQ^T N^T + \frac{1}{\gamma^2} I - (D^T D)^{-1} & MQ^T H \\ HQM^T & -I \end{bmatrix} \le 0. \tag{14}$$

The matrices $M(q)$ and $N(q,\dot{q})$ are the nonlinear function of q and $\dot{q}$ in Eq. (14). However, those matrices include trigonometric functions and can be bounded when each joint velocity range between two empirically determined extreme values. Using this fact, we suppose that the matrices forming above NLMI vary in some bounded sets of the space of matrices, i.e.,

$$[M(q), N(q,\dot{q}), H, D] \in \mathrm{Co}\left\{[M_i, N_i, H, D]\,|_{i\in\{1,2,\cdots,L\}}\right\},$$

where Co represents the convex hull.

Therefore, if

$$\begin{bmatrix} -N_i QM_i^T - M_i Q^T N_i^T + \frac{1}{\gamma^2} I - (D^T D)^{-1} & M_i Q^T H \\ HQM_i^T & -I \end{bmatrix} \le 0$$

have a common solution Q for all $i \in \{1, 2, \cdots, L\}$ then Q is also a solution to Eq. (14) and the stabilizing control input is determined as

$$u = -\left(D^T D\right)^{-1} G_2^T Q^{-1} s.$$

This approach provides tractable method to get constant solutions to NLMI, which can be used to design the control input. However, this approach generally leads to conservative results if the prescribed bound is large.

3.3 Real Control Input

The total control input at the torque level becomes

$$\begin{aligned} K &= \hat{M}(q)\ddot{q}_r + \hat{N}(q,\dot{q})\dot{q}_r + \hat{G}(q) - (D^T D)^{-1} G_2^T Q^{-1} s \\ &= K_\tau (i_{w1} + i_{w2}) \\ &= K_\tau i_w. \end{aligned}$$

Using $y = i - i_w$ as a new coordinate, we rewrite

$$\begin{aligned} \dot{s} &= -M^{-1}(q)N(q,\dot{q})s + M^{-1}(q)w \\ &\quad + M^{-1}(q)K_\tau i_{w2} + M^{-1}(q)K_\tau y \\ \dot{y} &= -L_m^{-1} R_m i - L_m^{-1} K_m \dot{q} + L_m^{-1} v - \frac{di_w}{dt}. \end{aligned}$$

To design the robust control, which achieves L_2-gain property, we use the positive definite storage function

$$V(s,y) = E(s) + \frac{1}{2} y^T y.$$

Its time derivative is

$$\dot{V} \leq \gamma^2 \|w\|^2 - \|z\|^2 + y^T \left(\Delta A + L_m^{-1} v\right)$$

where $\Delta A = 2K_\tau M^{-T} P s - L_m^{-1} R_m i - L_m^{-1} K_m \dot{q} - \frac{di_w}{dt}$.

ΔA can be bounded as

$$\begin{aligned} \|\Delta A\| &\leq \alpha_1 \|s\| + \alpha_2 \|i\| + \alpha_3 \|\dot{q}\| + \left\| \frac{di_w}{dt} \right\| \\ &:= \rho\left(s, i, \dot{q}, \frac{di_w}{dt}\right). \end{aligned}$$

If it is assumed that $\underline{L} \leq L \leq \overline{L}$ for known upper and lower bounding matrices, the control input can be chosen as

$$v = -\overline{L}\left(\frac{y}{\|y\|}\rho - y\right).$$

Table 1: Manipulator parameters used in the simulation

	Real Length	Real Mass	Bound of Mass
Link 1	0.5 m	2 kg	[1.5,2.5]
Link 2	0.3 m	1 kg	[0.5,1.5]

With the designed control input, the storage function satisfies $\dot{V} \leq \gamma^2 \|w\|^2 - \left(\|z\|^2 + \beta \|y\|^2\right)$, $\beta > 0$ which achieves L_2-gain property. To smooth out the control discontinuity, the saturation-type control can be chosen as

$$v = -\overline{L}\left(\frac{y}{\|y\| + \varepsilon \cdot \exp(-\delta \mathrm{t})}\rho - y\right)$$

where ε and δ are positive constants.

4 Simulation

Robust control using recursive method is designed for two dof planar robot manipulator with actuators. Simulation was performed under parameter uncertainties. The objective of the simulation is to show the enhancement of robustness to parameter uncertainty. The set of dynamic parameter is summarized in Table 1 and 2. As an extreme disturbance, the mass of link 2 is assumed to vary by 50% at 2 second. The system model matrices forming LMIs are determined by the bound of parameter uncertainty and the trigonometric functions. The LMIs for the matrix Q are solved using an efficient convex algorithm in Matlab toolbox. It should be noted that the ease of controller tuning can be obtained since the solution of LMIs, if any, is found easily by an optimization algorithm.

The joints of manipulator are commanded to trace trajectories shown in Fig. 1(a) with some initial errors. The initial errors of the joints are $11.45°$ and $17.19°$, respectively. The estimates of the manipulator model matrices in Eq. (11) are assumed to be $\hat{M} = 0$ and $\hat{N} = 0$. The estimate of the gravity torque G is determined from the equation in the dynamics using the estimates of mass $\hat{m}_1 = 1.8$ kg and $\hat{m}_2 = 0.8$ kg.

The position error and input voltage are shown in Fig. 1(b) & 2, rspectively. Though the control input is saturated at the initial state, the proposed controller shows satisfactory robustness performance even under the large parameter uncertainty.

Table 2: Actuator parameters used in the simulation

	Real Value	Bound
Inductance	0.05 H	[0.03,0.07]
Resistance	0.3 Ω	[0.2,0.5]
Back emf Constant	0.065 Vsec/rad	[0.055,0.075]
Torque Constant	2 Nm/A	[1.9,2.1]

5 Conclusion

Using recursive design, robust control is designed for nonlinear uncertain systems, which can be decomposed into two cascaded subsystem. First, fictitious robust controller is designed using nonlinear H_∞ control. The associated HJ inequality is transformed to NLMI and its approximated solution is obtained from the fact that the terms in matrices can be bounded. The application of proposed method is simple since the gain matrix can be obtained easily by an efficient convex optimization algorithm. Second, the actual robust control is designed recursively through the design of fictitious control. The designed robust control is applied to robotic system with actuators.

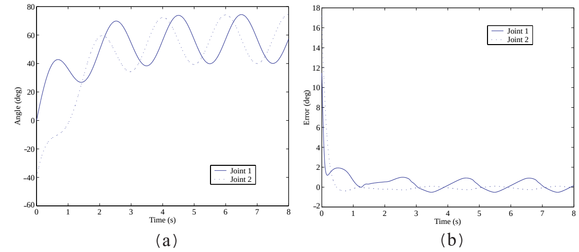

Figure 1: (a) Desired trajectories of joints (b) Position errors of joints

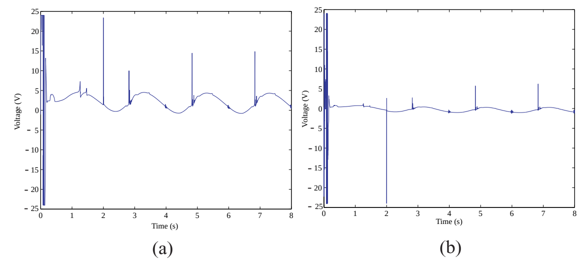

Figure 2: (a) Voltage of actuator of joint 1 (b) Voltage of actuator of joint 2

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