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An On-Line Trajectory Modifier for the Base Link of Biped Robots To Enhance Locomotion Stability

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Abstract

This paper proposes an on-line trajectory modification scheme for biped robots to cope with uncertainty of their environment. For stable locomotion, biped robots should have robust properties against various disturbances such as ground irregularity and external pushing or pulling forces. Since such uncertainty is not known a priori, biped robots should have the ability to adapt to them on-line. In the proposed scheme, the trajectory of the base link in the vertical direction is modified depending on the magnitude of ZMP deviation from its safety boundary such that appropriate angular moment is generated to maintain stable walking. The modified trajectory then gradually returns to the original trajectory using a 3rd- or 5th-order interpolation polynomial. And this paper expands the gravity-compensated inverted pendulum mode (GCIPM) to generate the base-link trajectory not only for single support phases but also double support phases. In the simulations to evaluate the proposed scheme, an impedance controller is used to control a 6-dof biped robot and the environment of the biped robot is assumed to consist of nonlinear and linear compliant contact models. The simulation results show that the proposed on-line trajectory modification scheme effectively enhances the stability of locomotion.

1 Introduction

Biped robots have better mobility potential than conventional wheeled mobile robots, especially for walking on rough terrain and climbing stairs. In order to be fully mobile, biped robots must deal with ground irregularity and maintain their dynamic stability. Moreover, since biped robots may experience various disturbances like pushing or pulling forces, they must have robust properties against such unknown disturbances for stable walking.

Many researchers have been working on the schemes to stabilize the walking of biped robots. For example, Huang et al. [1] proposed a method to plan a walking pattern consisting of a foot trajectory and a hip trajectory. They formulate the constraints of a foot trajectory and generate foot and hip trajectories with a high stability margin. Yamaguchi, Takanishi and his co-workers [2, 3, 4, 7] used trunk motions to realize stable walking for arbitrarily planned motion trajectory, known disturbances, and ZMP (Zero Moment Point) trajectory. Kajita and Tani [5] discussed an adaptive gait control of a biped robot based on real-time sensing of the ground profile with an ultrasonic sensor.

Using the ZMP equation and the information on the ZMP, Park and Chung [9] proposed an on-line trajectory algorithm to increase the stability robustness of locomotion. The algorithm modifies the desired acceleration of the base in its vertical direction to compensate the external moment that is caused by unexpected external forces applied to the biped robot. This algorithm is so simple as to be used on-line and requires the measurement or estimation on the acceleration of the swing leg as well as that of the base link.

If some disturbances are applied to the robot in the direction of its motion, its ZMP is suddenly increased and may become so large that it may go to the very end of the tip of its supporting leg before the biped robot falls down eventually. In [9], the trajectory modification algorithm is activated to increase the vertical acceleration of the base link only when the sensed ZMP goes out of its preset ZMP boundary, which is near the ends of the foot print. For this reason, this algorithm does *not* effectively compensate the trajectory when the ZMP very rapidly approaches to the preset boundary inside of its preset boundary. This observation motivates a new ZMP compensation algorithm, where the desired vertical acceleration of the base link is a function of deviation from the safety boundary. Once the trajectory of the base link is changed with the ZMP compensation algorithm, it must return to its original trajectory. Otherwise, a large change in the height of the base link may compromise the stability if the locomotion pattern generated based on the inverted pendulum mode or the gravity-compensated inverted pendulum mode (GCIPM) continues. This paper proposes a polynomial interpolation to generate a trajectory for the base link that merges smoothly with its original trajectory.

Kajita and his co-workers suggested the linear inverted pendulum mode (LIPM) to generate the trajectory of the base link of biped robots [6]. Since the biped locomotion generated by the LIPM results in large movements of the ZMP, Park and Kim [8] proposed the gravity-compensated inverted pendulum mode (GCIPM), which is based on the model of the biped robot that consists of two masses representing the swinging leg and the base link. The locomotion generated by the GCIPM in [8] has only single support phases. In this paper, a modified GCIPM is proposed to include both single and double support phases, and is used to generate the reference trajectory of the base in later simulations.

Section 2 describes the dynamics of biped robots. Section 3 derives the impedance controller for biped robots. Section 4 is devoted to the modified GCIPM and the proposed on-line modification of the trajectory of the base link. Simulations of the proposed scheme for a biped robot and their results are shown in Section 5, followed by conclusions in Section 6.

2 Dynamics of Biped Robots

The biped robot considered in this paper has 3 degrees of freedom in each leg. Its dynamics is described by

$$\begin{aligned} H_c\ddot{q}_c + G_c a_0 + D_c h_c + n_c &= \tau_c && (1)\\ H_u\ddot{q}_u + G_u a_0 + D_u h_u + n_u &= \tau_u && (2)\\ Q_c\ddot{q}_c + Q_u\ddot{q}_u + R a_0 + P_c h_c + g &= 0 && (3)\end{aligned}$$

where $\ddot{q} \in \mathbb{R}^3$, $a_0 \in \mathbb{R}^6$, and $h \in \mathbb{R}^6$ are the joint acceleration, the acceleration of the base link, and the force applied at the feet by the ground, respectively; $H \in \mathbb{R}^{3\times3}$, $G \in \mathbb{R}^{3\times6}$, $Q \in \mathbb{R}^{6\times3}$, and $R \in \mathbb{R}^{6\times6}$ are the inertia-related matrices; $D \in \mathbb{R}^{3\times6}$ is a Jacobian matrix; $n \in \mathbb{R}^3$ and $g \in \mathbb{R}^6$ are the Coriolis and centripetal term, and gravitational term; $\tau \in \mathbb{R}^3$ is the joint torque.

Equations (1) and (2) describe the dynamics of the supporting and free legs respectively, and Eq. (3) describes the dynamics of the base link coupled with the legs. Subscripts 'c' and 'u' denote 'constrained' and 'unconstrained', respectively. Note that the external force at the foot of the unconstrained leg, h_u, becomes zero when it moves into its swing phase.

3 Impedance Control for Supporting and Free Legs

In this paper, the impedance controller proposed in [10] is used, and thus summarized here.

Suppose that the desired impedance of the unconstrained leg is expressed by

$$M_u\ddot{e}_{ue} + B_u\dot{e}_{ue} + K_u e_{ue} = f_0 - f, \qquad (4)$$

where $e_{ue} = x_{ue} - x_{ue,d}$ and subscript 'd' denotes the 'desired' value; M_u, B_u, and K_u are the desired mass, damping coefficient, and stiffness; f is the resultant external force; reference force f_0 is selected as

$$f_0 = \begin{cases} \begin{bmatrix} 0_{2\times1} \\ K_u t_{pad} + w_t \\ 0_{3\times1} \end{bmatrix} & \text{when the pad is squeezed,} \\ 0_{6\times1} & \text{otherwise} \end{cases}$$

where w_t is the target weight, which helps the swinging leg take some of the weight of the biped robot so that it can proceed to the double support phase. With the impedance model, the control input for the unconstrained leg becomes

$$\begin{aligned}\tau_u = H_u J_{ue}^{-1}\big[&-M_u^{-1}B_u(\dot{x}_{ue} - \dot{x}_{ue,d}) \\ &- M_u^{-1}K_u(x_{ue} - x_{ue,d}) + M_u^{-1}(f_0 - f) - a_0 \\ &- \dot{J}_{ue}\dot{q}_u\big] + G_u a_0 + D_u h_u + n_u. \end{aligned} \qquad (5)$$

Similarly, suppose that the desired impedance model for the constrained (supporting) leg is selected as

$$M_0\ddot{e}_0 + B_0\dot{e}_0 + K_0 e_0 = 0, \qquad (6)$$

where $e_0 = x_0 - x_{0,d}$ and subscript '0' denotes the base link; M_0, B_0, and K_0 are the desired mass, damping, and stiffness, respectively. Then, the control input for the constrained leg, τ_c, is described as

$$\tau_c = (Q_c H_c^{-1})^{-1}[\tilde{R}\{a_{0,d} - M_0^{-1}\{B_0(\dot{x}_0 - \dot{x}_{0,d}) + K_0(x_0 - x_{0,d})\}\} - \tilde{P}h - \tilde{g}], \quad (7)$$

where

$$\tilde{R} = Q_c H_c^{-1} G_c + Q_u H_u^{-1} G_u - R$$
$$\tilde{g} = g + Q_u H_u^{-1}(\tau_u - n_u) - Q_c H_c^{-1} n_c$$
$$\tilde{P} = \left[P_c - Q_c H_c^{-1} D_c \quad P_u - Q_u H_u^{-1} D_u\right]$$
$$h = \left[h_c \quad h_u\right]^T$$

4 Trajectory Generation of Base Link

4.1 Modified GCIPM

The GCIPM is based on a simple biped model which consists of two particles, one for the base link and the other for the free leg. It is from the observation that the most weight of the biped robot is concentrated around its base link and that the the free leg sometimes in locomotion moves a far from the base link, its inertia becoming significant. Figure 1 shows the two-particle model for the GCIPM, where M and m denote the mass of the base link and the free leg, respectively.

Considering the Y-directional moment about the ZMP, the following equation is obtained:

$$\ddot{X}(t) - \omega^2 X(t) = F(t), \quad (8)$$

where $F(t) = \beta(gx - z\ddot{x} + x\ddot{z})$, $\beta = m/(MH_z)$ and $\omega = \sqrt{g/H_z}$; $X(t)$ and $x(t)$ denote the trajectory of the base link and the free leg, respectively.

To derive Eq. 8, it is assumed that the robot moves only in the sagittal plane and the height of the base link remains constant at $z = H_z$. As far as the free leg is concerned, any appropriate trajectory can be selected. In this paper, the following trajectory is selected for the free leg.

$$x(t) = \begin{cases} -S\cos(\omega_f t) & 0 \le t \le T_s \\ S & T_s < t \le T \end{cases}$$

$$z(t) = \begin{cases} 0.5h_f\left[1 - \cos(2\omega_f t)\right] & 0 \le t \le T_s \\ 0 & T_s < t \le T \end{cases}$$

where S and h_f are the stride and the maximum foot height, respectively; T and T_s are the time span of a single step and a single support phase, respectively; $\omega_f = \pi/T_s$.

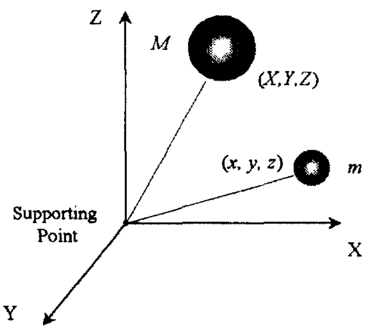

Figure 1: A simple two-particle model for the modified GCIPM.

Given the trajectory of the free leg above, it is too complicated to solve $F(t)$ analytically. However, under the assumption that T_s and h_f are selected such that $\omega_f^2 h_f \ll g$, $F(t)$ can be approximated by a simple function:

$$F(t) \approx \begin{cases} -\beta g S\cos(\omega_f t) & 0 \le t \le T_s \\ \beta g S & T_s < t \le T \end{cases}$$

Then, solving Eq. (8) results in

$$X_S(t) = C_1 e^{\omega t} + C_2 e^{-\omega t} + \eta\cos(\omega_f t) \quad (9)$$
$$X_D(t) = C_3 e^{\omega t} + C_4 e^{-\omega t} - \beta g S/\omega^2, \quad (10)$$

where $\eta = \beta g S/(\omega^2 + \omega_f^2)$ and 'S' and 'D' denotes 'single-support' and 'double-support', respectively.

Coefficients C_i can be determined by the initial conditions, i.e., $X_S(0)$ and $\dot{X}_S(0)$, and the continuity condition of the position and velocity at $t = T_s$, i.e, $X_S(T_s) = X_D(T_s)$ and $\dot{X}_S(T_s) = \dot{X}_D(T_s)$. Thus,

$$C_1 = 0.5\left(X_S(0) + \frac{1}{\omega}\dot{X}_S(0) - \eta\right)$$
$$C_2 = 0.5\left(X_S(0) - \frac{1}{\omega}\dot{X}_S(0) - \eta\right)$$
$$C_3 = C_1 - 0.5\left[\eta e^{-\omega T_s} - \frac{\beta g S}{\omega^2} e^{-\omega T_s}\right]$$
$$C_4 = C_2 - 0.5\left[\eta e^{\omega T_s} - \frac{\beta g S}{\omega^2} e^{\omega T_s}\right]$$

Since the trajectory should be repeatable for each step, two additional conditions, i.e., $X_S(0) = -X_D(T)$ and $\dot{X}_S(0) = \dot{X}_D(T)$, should be satisfied. Using these conditions, the initial velocity of the base, $\dot{X}_S(0)$, can be represented by $X_S(0)$:

$$\dot{X}_S(0) = \omega\left(X_S(0) - \eta\right)\frac{1 + e^{\omega T}}{1 - e^{\omega T}} + \left(\frac{\omega\eta - \beta g S}{\omega}\right)\frac{e^{\omega(T - T_s)} + e^{-\omega(T - T_s)} - 2}{e^{\omega T} - e^{-\omega T}}$$

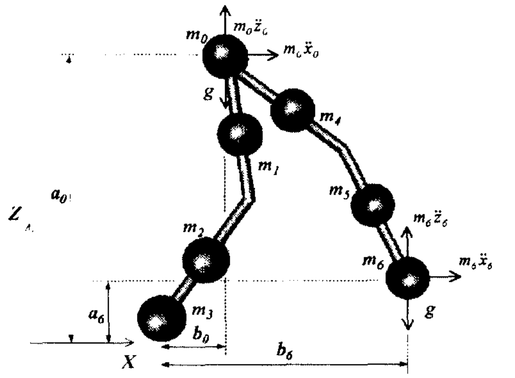

Figure 2: The particle model of a biped robot.

Note that when the locomotion consists of only single support phases, i.e., $T_s = T$, the above result becomes identical to that of [8]. Moreover, under the assumption that the mass of the free leg, m, is zero, it becomes identical to that of the LIPM.

4.2 On-Line ZMP Compensation

A base trajectory is generated to keep the ZMP in the stable region. Suppose that the biped robot consists of 7 particles as in Fig. 2. Since the robot walks in the sagittal plane, only the ZMP in the moving direction, x_{zmp}, is important in investigating its locomotion stability. In a single support phase, the ZMP in the X-direction is written as:

$$x_{zmp} = \frac{\sum_i M_i}{\sum_i F_i} = \frac{\sum_{i=0}^{6} m_i(\ddot{z}_i + g)x_i - \sum_{i=0}^{6} m_i \ddot{x}_i z_i}{\sum_{i=0}^{6} m_i(\ddot{z}_i + g)} \tag{11}$$

where $\sum_i M_i$ and $\sum_i F_i$ represent the resultant moment about the ZMP and the resultant force exerted at the foot from the ground, respectively, and g denotes the gravity.

From the equation it is easily observed that at any moment the size of $\sum_i M_i$ should be reduced and/or the size of $\sum_i F_i$ should be increased in order to keep the size of the ZMP smaller. Based on this, the following control strategy to compensate any external adversary moment is decided:

- Reduce the size of b_0 and/or b_6 if it is necessary to move the ZMP toward the center of the foot.
- Increase (or decrease) $\ddot{x}_0$ and/or $\ddot{x}_6$ if the ZMP should move backward (or forward).
- Increase $\ddot{z}_0$ and/or $\ddot{z}_6$ to the vertical direction if it is necessary to move the ZMP toward the center of the foot.

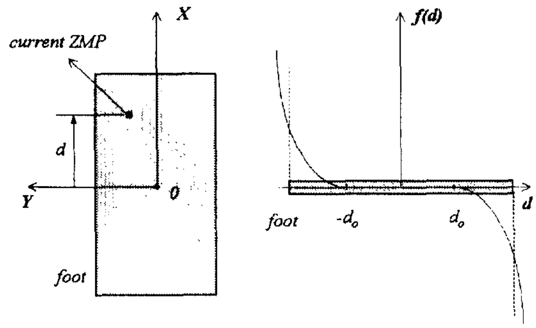

Figure 3: Definitions of d and $f(d)$.

If the 1st or 2nd strategy is implemented, $\ddot{x}_i$ and x_i in the predefined walking pattern should be modified. If they are modified, the stride of locomotion is also changed, and then the other trajectories such as the vertical direction trajectories of base and the foot must be regenerated based on the new value of the stride. On the other hand, $\ddot{z}_i$ and z_i can be changed with relatively little effects on the original locomotion pattern. For this reason, in this paper, the last strategy is used for the ZMP compensation.

Differentiating Eq. (11) with respect to $\ddot{z}_j$,

$$\frac{\partial x_{zmp}}{\partial \ddot{z}_j} = \frac{m_j}{\left(\sum_i m_i(\ddot{z}_i + g)\right)^2}\left[x_j \sum_i F_i - \sum_i M_i\right] \tag{12}$$

The effect of $\ddot{z}_j$ on x_{zmp} is larger as m_j is larger. As far as the acceleration of the base link, $\ddot{z}_0$, is concerned, its effect depends on the sign of $[x_j \sum_i F_i - \sum_i M_i]$. For example, a positive (negative) value of $\ddot{z}_0$ would increase (decrease) x_{zmp} when $[x_j \sum_i F_i - \sum_i M_i] > 0$. Thus, as long as $[x_j \sum_i F_i - \sum_i M_i] > 0$, $\ddot{z}_0$ should have a positive (negative) value when the ZMP is too near the heel (toe). In this paper, the desired vertical acceleration of the base link, $\ddot{z}_{0,d}$, is set depending on the ZMP deviation from its nominal position, i.e., the center of the foot:

$$\ddot{z}_{0,d} = \begin{cases} f(d) & [x_j \sum_i F_i - \sum_i M_i] > 0 \\ -f(d) & \text{otherwise} \end{cases} \tag{13}$$

where d is the deviation of the ZMP from the center of the foot, and $f(d)$ is a function based on a 2nd-order polynomial, as shown in Figure 3.

Current ZMP is only required to implement this scheme. Note that as long as the ZMP is not deviated too much from the center of the foot, i.e., $|d| < d_o$, where d_0 is a constant, the trajectory modifier is not activated. However, when the ZMP goes out of $\pm d_0$

approaches to the safety boundary due to some external disturbances, it is activated. Since the ZMP can be simply calculated with the measurement of the total reaction force, $\sum_i F_i$, and moment, $\sum_i M_i$ at the supporting foot, this scheme can be implemented on-line.

Once $\ddot{z}_0$ is increased, the height of the base, z_0, deviates from its original one. Such a height change, if left for a long period of time, could compromise the stability of locomotion since it does not satisfy the conditions used in the trajectory planning. To prevent this, the height of the base link must return to its original value soon after the modifier is trigger. In this paper, a 3rd- or 5th-order interpolating polynomial is used to generate the returning path.

5 Simulation

Performance of the proposed scheme is investigated and the effectiveness of the returning path based on the 3rd- and 5th-order polynomial are compared in simulations. In order to provide more realistic forces from the ground, the 3-dof environment model proposed in [8, 9] is used.

Figure 4 shows ZMP trajectory when there is no external disturbance force during the locomotion. Although the ZMP oscillates severely when a swinging leg contacts with ground, it still remains within its safety region. Then, the biped robot is pushed behind by a force of 15 N at the center of the base link for the duration of 0.2 s (from t=1.8–2.0 s). Figure 5 shows that the biped robot with the on-line compensation has a larger stability margin than that without the compensation. Note that the ZMP nearly reaches the toe when no compensation is implemented. The foot length of the robot is 20 cm.

When the height of the base reaches at a specific value with the ZMP compensation, the returning trajectory generator is activated. In Fig. 5, it can be observed that oscillations in ZMP with 5th-order polynomial dies out more quickly than with 3rd-order case. This is because that continuous desired acceleration of the base cannot be obtained with 3rd-order polynomial returning path (Figure 6). This sudden changes in desired base acceleration cause oscillations of ZMP trajectory, and effect on robot stability.

Even though the 5th-order return path guarantees a smoother change in acceleration, it generates larger $z_{0,d}$ than 3rd-order case does, as can be shown in Fig. 7. Too large z_0 cannot be realizable since there is a limit in the length of a leg. Therefore, the 3rd-order return path is more desirable in the case of the presence of a large amount of disturbance.

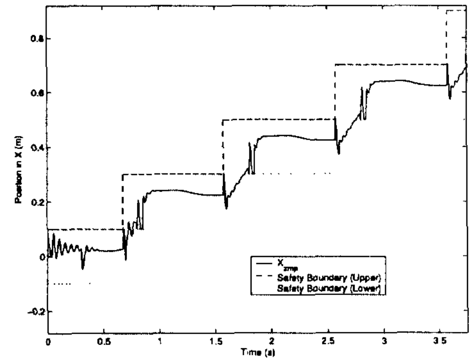

Figure 4: ZMP trajectory without disturbance.

When the external disturbance force is applied, the left leg of the biped robot is in its swing phase. With the proposed on-line trajectory generator, the height of the base link increases up to about 1.5 cm; however, it returns to its nominal level smoothly as in Fig. 8.

6 Conclusions

An on-line trajectory modifier is proposed to cope with some external disturbance to biped robots during their locomotion. And, a modified GCIPM, which expands the previous GCIPM to cover locomotion with double support phases, is derived. A series of simulations show that the biped robot with the proposed trajectory generator can walk even when there is a certain amount of external disturbance force. A 5th-order polynomial for return paths generates a smooth ZMP trajectory; however, it is less desirable than a 3rd-order polynomial when there exists a large amount of disturbance due to too much elevation of the base link. The on-line ZMP compensation scheme proposed here is a bit limited in the sense that only the vertical trajectory of the base link is modified. However, it is highly desirable to come up with a some kind of on-line stride generation scheme in near future.

References

[1] Q. Huang et al., "A High Stability, Smooth Walking Pattern for a Biped Robot," Proc. Int. Conf. on Rob. and Autom., pp. 65–71, 1999.

[2] J. Yamaguchi and A. Takanishi, "Development of a Leg Part of a Humanoid Robot-Development

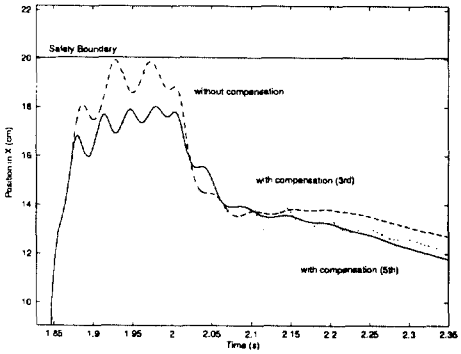

Figure 5: ZMP trajectories when the robot is pushed behind by a force of 15 N.

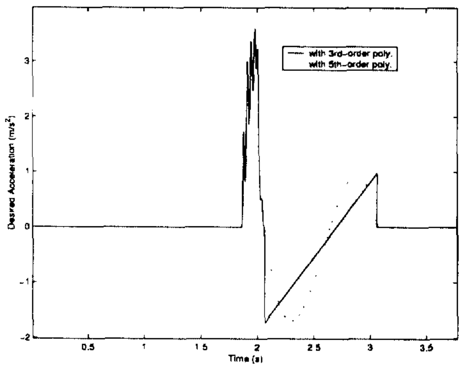

Figure 6: Desired accelerations of the base link when the robot is pushed behind by a force of 15 N.

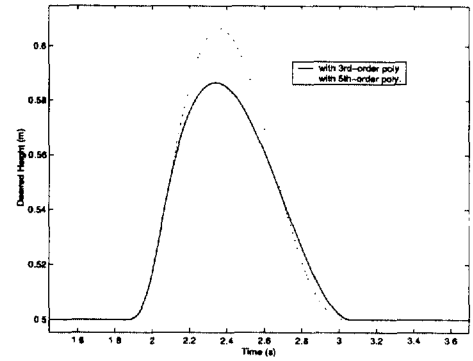

Figure 7: Modifications of the desired height of the base link.

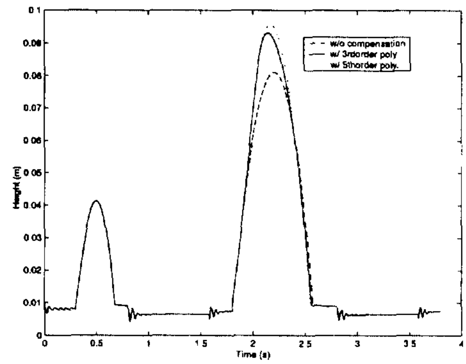

Figure 8: Trajectory of the left foot.

of a Biped Walking Robot Adapting to the Humans' Normal Living Floor," Autonomous Robots, pp. 369-385, 1997.

[3] J. Yamaguchi et al., "Development of a Dynamic Biped Walking System for Humanoid-Development of a Biped Walking Robot Adapting to the Humans' Normal Living Floor," Proc. Int. Conf. on Rob. and Autom., pp. 232–239, 1996.

[4] J. Yamaguchi, A. Takanishi, and I. Kato, "Development of a Biped Walking Robot Compensating for Three-Axis Moment by Trunk Motion ," Proc. Int. Conf. on Intelligent Rob. and Syst., pp. 561–566, 1993.

[5] S. Kajita and K. Tani, "Adaptive Gait Control of a Biped Robot based on Realtime Sensing of the Ground Profile," Proc. Int. Conf. on Rob. and Autom., pp. 570–577, 1996.

[6] S. Kajita, T. Yamaura, and A. Kobayashi, "Dynamic Walking Control of a Biped Robot Along a Potential Energy Conserving Orbit," IEEE Trans. on Rob. and Autom., pp. 431–438, 1992.

[7] A. Takanishi et al., "Dynamic Biped Walking Stabilized with Optimal Trunk and Waist Motion," IEEE/RSJ Int. Workshop on Rob. and Syst., pp. 187–192, 1989.

[8] J. H. Park and K. D. Kim, "Biped Robot Walking Using Gravity-Compensated Inverted Pendulum Mode and Computed Torque Control," Proc. Int. Conf. on Rob. and Autom., pp. 3528–3533, 1998.

[9] J. H. Park and H. Chung, "ZMP Compensation by On-Line Trajectory Generatin for Biped Robots," Proc. Conf. on Syst., Man and Cyb., 1999.

[10] J. H. Park and H. Chung, "Impedance Control and Modulation for Stable Footing in Locomotion of Biped Robots," Proc. Int. Conf. on Intelligent Rob. and Syst., 1999.