Impedance Modulation for a Teleoperator
Using Distance Measurement

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Abstract

This paper proposes a new impedance control scheme based on a variable stiffness matrix for a bilateral teleoperation. In this scheme, stiffness matrix of the impedance model in the slave is modulated according to the distance, measured by an ultrasonic sensor, between the slave and environment. At the same time, the stiffness matrix of the master is also changed properly in order to maintain the impedance parameters of the combined system to be constant. The proposed scheme is implemented on a 1-dof master/slave system to perform a simple task. In the experiments, the teleoperator with the impedance parameter modulation shows better performance than one with constant impedance parameter, especially in the task execution time and the excessive external forces.

1 Introduction

Impedance control, proposed by Hogan [1], adjusts the impedance of the manipulator, which is defined as $Z(s) = F(s)/V(s)$ where $F(s)$ denotes force and $V(s)$ denotes velocities in the Laplace transform and is determined typically by an inertia, a damper, and a spring. The desired impedance of the manipulator depends on the task that the manipulator performs.

In many telerobotic tasks, robot manipulators interact with their environments. Excessive contact force between the robot and the environment should be prevented for the stability of the controlled system and avoiding system damages. Besides, the tracking ability in a freespace cannot be neglected for the task performance. Even in the same task, several situations produce conflicting choices of desired impedance parameters. For example, small stiffness can effectively reduce impact forces; on the other hand, large stiffness may improve the tracking performance in a freespace.

Many researchers applied the impedance control to a bilateral teleoperation system and adjust impedance parameters to the given task with the information of contact force and/or commanded velocities [2, 3, 4, 5, 6]. Dubey et al. [2, 3] used variable damping and stiffness for a compromise between the requirements of tracking and reduction of impact forces. Rubio et al. [4] proposed a control algorithm to adaptively vary the desired impedance in order to increase stability in hard contact tasks. Salcudean, Zaal and their colleagues suggested adaptive environment impedance estimation and applied this scheme to an excavator [5, 6]. And Funaya [7] also adapted the robot impedance as the environment impedance changes. Since these methods used contact force information for impedance modulation and/or adopted the adaptive control theory, they have the limitation that they cannot reduce a contact force rapidly at the first contact.

This paper presents a new algorithm to modulate the stiffness matrix of the impedance model according to the distance between the slave end-effector and the environment. Since the distance information is obtained with the ultrasonic sensor, this scheme is able to reduce contact forces more effectively than other methods, which used the contact force and position error information.

Experiments on a 1-dof teleoperation system were done by several trained operators to perform a simple task with fixed stiffness values as well as the modulated stiffness. The task completion time and the number of excessive contact force are used as performance indices.

Section 2 describes the dynamics of the bilateral teleoperation system and derives the impedance controller for the system. Section 3 summarizes the proposed impedance modulation scheme. The experiments and their results are shown in Section 4, followed by conclusions in Section 5.
Controller Design

In this section, impedance controllers for the master and the slave are derived. These controllers are designed based on a typical position-force teleoperation. The impedance-controlled master device reflects to the human operator the contact force between the slave and its environment according to the desired master impedance model. And the slave is controlled to follow the trajectories generated from the desired slave impedance model.

2.1 Dynamics of Master and Slave

The dynamics of the master/slave system are modeled as a mass-damper system as follows:

\[ M_m \ddot{\theta}_m + C_m (\theta_m, \dot{\theta}_m) = \tau_m + J_m^T F_H \]  
(1)

\[ M_s \ddot{\theta}_s + C_s (\theta_s, \dot{\theta}_s) = \tau_s - J_s^T F_E \]  
(2)

where \( \theta \in \mathbb{R}^{6 \times 1} \) and \( \tau \in \mathbb{R}^{6 \times 1} \) denote joint angle and input torque vectors; \( M \in \mathbb{R}^{6 \times 6} \) and \( C \in \mathbb{R}^{6 \times 1} \) denote mass and viscous coefficient matrices; \( F_H \in \mathbb{R}^{6 \times 1} \) is the force applied at the master by the operator, and \( F_E \in \mathbb{R}^{6 \times 1} \) is the force exerted on the slave by the environment; \( J \) is the Jacobian matrix; subscript ‘m’ and ‘s’ denote the master and the slave, respectively.

2.2 Impedance Control for a Teleoperation System

With the impedance control, the desired characteristics between the human force and the external force can be selected appropriately. First, suppose that the desired impedance in the world coordinates for the slave is specified by

\[ M_s \ddot{x}_s + B_s \dot{x}_s + C_s \dot{x}_s = F_E \]  
(3)

or

\[
\begin{bmatrix}
    M_d^d & 0_{3 \times 3} \\
    0_{3 \times 3} & M_d^r
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_s \\
    \ddot{x}_s
\end{bmatrix}
+ 
\begin{bmatrix}
    B_d^d & 0_{3 \times 3} \\
    0_{3 \times 3} & B_d^r
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_s \\
    \dot{x}_s
\end{bmatrix}
+ 
\begin{bmatrix}
    K_d^d & 0_{3 \times 3} \\
    0_{3 \times 3} & K_d^r
\end{bmatrix}
\begin{bmatrix}
    x_s \\
    x_s
\end{bmatrix}
= F_E
\]

where \( M, B \) and \( K \) are the desired inertia, damping coefficient, and stiffness matrix, respectively; superscript ‘d’ and subscript ‘r’ denote ‘desired’, ‘translation’, and ‘rotation’, respectively; \( x \) denotes the end effector position/orientation; \( x_{sd} := x_s - x_d \) and \( x_{sr} := x_s - x_{sr} \) are the difference between the current and desired position/orientation of the slave.

The desired impedance model for the master is similarly defined as follows.

\[ M_{md} \ddot{x}_m + B_{md} \dot{x}_m + K_{md} x_m = F_H - U_f \]  
(4)

or

\[
\begin{bmatrix}
    M_{md} & 0_{3 \times 3} \\
    0_{3 \times 3} & M_{mr}
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_m \\
    \ddot{x}_m
\end{bmatrix}
+ 
\begin{bmatrix}
    B_{md} & 0_{3 \times 3} \\
    0_{3 \times 3} & B_{mr}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_m \\
    \dot{x}_m
\end{bmatrix}
+ 
\begin{bmatrix}
    K_{md} & 0_{3 \times 3} \\
    0_{3 \times 3} & K_{mr}
\end{bmatrix}
\begin{bmatrix}
    x_m \\
    x_m
\end{bmatrix}
= F_H - U_f \]

where \( x_{md} := x_m - x_d \) and \( x_{mr} := x_m - x_{mr} \) are the difference between the current and desired position/orientation of the master; \( U_f \in \mathbb{R}^{6 \times 6} \) is the force scaling matrix.

Using the position scaling matrix, \( U_p \in \mathbb{R}^{6 \times 6} \), the position/orientation commands to the slave are modified such that

\[
\begin{bmatrix}
    x_{sd} \\
    x_{sr}
\end{bmatrix}
= U_p x_m = 
\begin{bmatrix}
    U_l & 0_{3 \times 3} \\
    0_{3 \times 3} & U_r
\end{bmatrix}
\begin{bmatrix}
    x_m \\
    x_m
\end{bmatrix}
\]

From Eqs. (4), (5) and (6), the following equation can be obtained

\[ \ddot{x}_{sd} - U_p M_{md}^{-1} B_{md} U_p^{-1} \dot{x}_{se} - U_p M_{md}^{-1} K_{md} U_p^{-1} x_{se} = U_p M_{md}^{-1} (F_H - U_f F_E) \]  
(7)

and, with Eqs. (3) and (7), the error dynamics of the slave part becomes:

\[ \ddot{x}_{se} + \ddot{B} \dot{x}_{se} + \ddot{K} x_{se} = M_{sd}^{-1} F_E - U_p M_{md}^{-1} (F_H - U_f F_E) \]  
(8)

where

\[
\begin{align*}
\ddot{B} &= M_{sd}^{-1} B_{sd} + U_p M_{md}^{-1} B_{md} U_p^{-1} \\
\ddot{K} &= M_{sd}^{-1} K_{sd} + U_p M_{md}^{-1} K_{md} U_p^{-1}.
\end{align*}
\]

If \( M, B \) and \( K \) are diagonal matrices, the above \( \ddot{B} \) and \( \ddot{K} \) can be simplified as follows:

\[
\begin{align*}
\ddot{B} &= M_{sd}^{-1} B_{sd} + M_{md}^{-1} B_{md} \\
\ddot{K} &= M_{sd}^{-1} K_{sd} + M_{md}^{-1} K_{md}.
\end{align*}
\]

Combining Eqs. (2) and (3) and removing acceleration term with \( \ddot{x}_s = J_s \dot{\theta}_s \) results in the impedance control input to the slave:
\[ \tau_s = -M_s J_s^{-1} \dot{J} \dot{\theta} + J_s^T F_E + C_s \left( \theta_s, \dot{\theta}_s \right) \\
+ M_s J_s^{-1} M_{sd}^{-1} \left( F_E - B_s \ddot{x}_s - K_{sd} \dot{x}_s \right). \tag{9} \]

Similarly, combining Eqs. (1) and (4) with the Jacobian matrix gives the impedance control input for the master.

\[
\tau_m = -M_m J_m^{-1} J_m \dot{\theta}_m - J_m^T F_H + C_m \left( \theta_m, \dot{\theta}_m \right) \\
+ M_m J_m^{-1} M_{md}^{-1} \left\{ F_H - U_f F_E \right\} \\
- B_{md} \ddot{x}_{me} - K_{md} \dot{x}_{me}. \tag{10} \]

3 Impedance Parameter Modulation

The master/slave teleoperation systems have to cope with the unconstrained and constrained conditions in order to perform a given task well. And, there is a tradeoff between the tracking performance in a freespace and the contact stability in selecting impedance parameters of the system. In other words, if the slave tracks well in a freespace with a certain impedance parameter, it is easy to become unstable in a hard contact case with the same parameter. On the other hand, if the system can reduce the contact force effectively with some parameters, the parameters may deteriorate the tracking performance in a freespace.

3.1 Variable Stiffness Matrix with Distance Measurement

In this section, the stiffness matrix of the slave is modified according to the distance between the slave and environment. If the slave moves far apart from the environment, the large stiffness in the impedance model will increase the tracking performance. Otherwise, a small stiffness matrix is used as the desired model in order to decrease the contact forces with the environment. When a contact occurs, the impedance parameters are set to be constant in order not to confuse the human operator’s judgement of a contact state.

The stiffness matrices for the master and slave will be expressed in the motion coordinate, where the first column of the motion coordinate is oriented along the command velocity vector. And, only translation component in the stiffness matrix is considered. First, the stiffness matrix of the slave is modified according to the distance as follows:

\[
K_{sd}^d = \begin{bmatrix} \tilde{k}_s & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_s \end{bmatrix} \tag{11} \]

Figure 1: Modulated stiffness of the master and slave.

where

\[
\tilde{k}_s = \begin{cases} k_{\text{min}, s} & d < 0 \\ \frac{a_1 + a_2 \cos \left( \frac{d}{d_{\text{max}}} \right)}{k_s} & 0 \leq d \leq d_{\text{max}} \\ k_s & d > d_{\text{max}} \end{cases} \]

with

\[
a_1 = \frac{1}{k_s}, \quad a_2 = \frac{k_s - k_{\text{min}, s}}{k_s k_{\text{min}, s}}\]

In the above equation, \( d \) is the minimum distance between the end effector of the slave and the environment measured with respect to the moving direction; \( d_{\text{max}} \) is a modulation range; \( k_{\text{min}, s} \) and \( k_s \) are the optimized gains for the contact and the freespace, respectively.

3.2 Stability of the Entire System

One way to maintain the entire system to be stable is to make the impedance parameters of the combined system, Eq. (8), be constant. If we choose the translation component of the desired inertia matrices to be:

\[ M_{sl}^d = m_s I_3 \quad \text{and} \quad M_{md}^d = m_m I_3, \]

then the translation component, \( \tilde{K}_t \in \mathbb{R}^{3 \times 3} \), of the combined stiffness matrix in Eq. (8) becomes:

\[
\tilde{K}_t = \frac{1}{m_s} K_{sl}^d + \frac{1}{m_m} K_{md}^d. \tag{12} \]

Using this relationship, the stiffness matrix of the master is modified in order to make the combined stiffness be constant as follows:

\[
K_{md}^d = \begin{bmatrix} \tilde{k}_m & 0 & 0 \\ 0 & k_m & 0 \\ 0 & 0 & k_m \end{bmatrix} \tag{13} \]
where

\[
\kappa_m = \begin{cases} 
\frac{m_m}{m_s} (k_s - k_{\text{min}, s}) + k_m & d < 0 \\
\frac{m_m}{m_s} \left( k_s - \frac{1}{a_1 + a_2 \cos \left( \frac{1}{ \omega_{\text{max}} d} \right) } \right) + k_m & 0 \leq d \leq d_{\text{max}} \\
k_m & d > d_{\text{max}}
\end{cases}
\]

With Eq. (13), the combined stiffness has a constant value of \(\frac{1}{m_s} k_s + \frac{1}{m_m} k_m\) independently of the distance, \(d\), where \(k_m\) denotes the optimized gain for the freespace. In a design procedure, \(k_s, k_{\text{min}, s}\) and \(k_m\) must be selected considering the task performance. Figure 1 shows \(k_m\) and \(k_s\) modulated according to the distance, \(d\).

4 Experiments

In this section, the performance of the proposed control scheme is investigated through the experiments with a 1-dof master/slave bilateral teleoperation system.

4.1 Structure of Teleoperation System

The master and the slave are driven by AC motors as shown in Fig. 2. At the master side, its human operator pulls and pushes a knob attached to its motor and the knob turns around the motor axis. A load-cell is installed between the motor axis and the knob and measures the force exerted at the master. At the other side, the slave can move straight back and forth with a ball-screw mechanism and a load-cell is installed at its tip, which measures the contact force exerted by the environment. The slave is also equipped with the ultrasonic sensor to measure the distance between the slave tip and the wall. In the experiments, a wall with a some compliance (about 31,000 [N/m]) is used as the environment of the slave. The detailed diagram of the entire system is given in Fig. 3.

The dynamics of the system and control inputs \((u_m, u_s)\) for the 1-dof teleoperation system are summarized below.

Figure 2: The master and slave system used in this experiment.

![Diagram of Master and Slave Systems]

Figure 3: A detailed description of the master/slave system.

4.2 Task Definition

In this experiment, a human operator manipulates the master device to bring the slave into contact with the environment. The desired task is just to keep the contact force in the range of 8 ~ 15 N for more 3 seconds. If the contact force reaches at the value of 8 N, a notice message appears on the screen to inform the operator that the contact force enters into the desired region. And, if the contact force exceeds 15 N, that trial is regarded as a failure with a warning message. The operator must try to make a contact continuously in order to accomplish one task until he/she succeeds ten times. The task execution time is defined as the time for the 10 times task completion. And the number of times of excessive contact forces is recorded for the performance comparison between several stiffness gains.

4.3 Experimental Results

Impedance controller with constant or modulated stiffness is implemented for the performance comparison. As the constant gain, \(k_s\) is set to be 50, 100, 150, and 200 [N/m]. And \(k_m\) is modulated from 50 to 200 [N/m] for the modulated case.
For this experiment, several testers are employed. They are asked to accomplish this task as soon as possible, and they tried many times this task before the real experiment in order to be familiar with the teleoperation system. They execute tasks with various stiffness (fixed or modulated case) in random order more than 10 times. During the experiment, they have no information of which gain is used for now.

Figures 4 and 5 show that the average and its deviation of the task completion time and the number of failure for each stiffness. The modulated stiffness case shows the best performance in this experiment. The proposed scheme uses the fully reduced stiffness at the vicinity of the wall, so the number of failure is relatively small comparing with the other cases. Though the case of $k_s = 50$ has also small value, the slave becomes sluggish in the workspace with this stiffness. So, some operators rush the master device to speed up the slave, and this results in the excessive external forces.

**5 Conclusions**

A new impedance control scheme based on a variable stiffness matrix for a bilateral teleoperation is proposed. The translation stiffness component of the impedance model is modulated based on the distance, measured by an ultrasonic sensor, between the slave and environment. And, the stiffness matrix in the master is also modified in order to keep the impedance parameters of the combined system constant. The proposed scheme is implemented on a 1-dof master/slave system to perform a simple task. In the experiments, the teleoperator with the impedance parameter modulation performs better than that with a fixed impedance parameter, especially in the task execution time and avoiding excessive contact forces.

**References**


